9. [8 points] Determine whether the following series converges or diverges. If it converges, determine if it is absolute or conditional convergence **Circle your final answer choice**. Fully justify your answer including using proper notation and showing mechanics of any tests you use.

$$\sum_{n=1}^{\infty} \frac{n(-2)^n}{3^n}$$

Circle one: **Absolutely Converges** **Conditionally Converges** **Diverges**

Solution: We use the ratio test to show absolute convergence. Let $a_n = \dfrac{(-1)^n(n^{1/2}+4)}{3n^{3/2}+2}$. Then, we get

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)(-2)^{n+1}}{3^{n+1}}\cdot\frac{3^n}{n(-2)^n}\right|$$

Since we take absolute value, we can drop our negative signs. Then, we can regroup terms and simplify :

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)}{n}\cdot\frac{2^{n+1}}{2^n}\cdot\frac{3^n}{3^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{(n+1)}{n}\cdot\frac{2}{1}\cdot\frac{1}{3}\right|$$

All of our terms are positive, so we can drop the absolute value signs. Therefore, we see that the limit is

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{2}{3}\lim_{n\to\infty}\frac{(n+1)}{n} = \frac{2}{3}$$

where one can conclude $\lim\limits_{n\to\infty}\dfrac{(n+1)}{n} = 1$ by L'Hopital, a dominating functions argument, or by writing $\frac{(n+1)}{n} = 1 + \frac{1}{n}$. Since $\frac{2}{3} < 1$, this series converges absolutely by the ratio test.