2. [14 points] Consider the following sequences, all defined for $n = 1, 2, 3, \ldots$

$$a_n = \int_0^n 10e^{-t}\,dt$$
$$b_n = (-1)^n \frac{100}{n^{0.75}}$$
$$c_n = 5(-3)^{n-3}$$

a. [3 points] Which sequences are monotone? No justification is required for this part of the problem. Circle your final answer(s) below.

Circle your answer(s): $\boxed{a_n}$ b_n c_n NONE

b. [3 points] Which sequences are bounded? No justification is required for this part of the problem. Circle your final answer(s) below.

Circle your answer(s): $\boxed{a_n}$ $\boxed{b_n}$ c_n NONE

c. [3 points] Which sequences are convergent? No justification is required for this part of the problem. Circle your final answer(s) below.

Circle your answer(s): $\boxed{a_n}$ $\boxed{b_n}$ c_n NONE

d. [5 points] Write a closed form expression for the series $\sum_{n=2}^{2023} c_n$. Your expression should be able to be evaluated using a simple calculator (i.e. no letters, no ellipses $(\ldots)$ and no sigma notation). Do not simplify the numbers in your expression.

Solution:

- The first term in the series is $c_2 = 5(-3)^{2-3} = -\frac{5}{3}$.
- The number of terms in the series is 2022.
- The common ratio between consecutive terms in the series is -3.

Using these three facts, and the formula for the sum of a finite geometric series, we obtain the answer below.

Answer: $\displaystyle\sum_{n=2}^{2023} c_n = -\frac{5}{3}\left(\frac{1-(-3)^{2022}}{1-(-3)}\right)$