3. [12 points] Consider the following sequences, all defined for $n = 1, 2, 3, \ldots$

$$a_n = \int_0^n 10e^{-t}\,dt$$
$$b_n = (-1)^n \frac{100}{n^{0.75}}$$

These are the same first two sequences from the previous problem.

a. [6 points] Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge? Fully justify your answer, including full mechanics of any tests you use.

Circle one: **Converges** $\boxed{\textbf{Diverges}}$

Solution: Consider

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \int_0^n 10e^{-t}\,dt = \lim_{n\to\infty} -10e^{-t}\Big|_0^n = \lim_{n\to\infty} -10e^{-n} + 10 = 10 \neq 0.$$

Therefore, by the Divergence Test, $\sum_{n=1}^{\infty} a_n$ must diverge.

b. [6 points] Does the series $\sum_{n=1}^{\infty} b_n$ converge or diverge? Fully justify your answer, including full mechanics of any tests you use.

Circle one: $\boxed{\textbf{Converges}}$ **Diverges**

Solution: Consider $d_n = \frac{100}{n^{0.75}}$. Since $d_n > 0$ (i.e. d_n is positive), and $d_{n+1} = \frac{100}{(n+1)^{0.75}} < \frac{100}{n^{0.75}} = d_n$ (d_n is decreasing) for all n, and

$$\lim_{n\to\infty} d_n = \frac{100}{n^{0.75}} = 0,$$

by the Alternating Series Test, we must have that $\sum_{n=1}^{\infty} (-1)^n d_n = \sum_{n=1}^{\infty} b_n$ must converge.