5. [8 points] Use the **comparison test** for series to determine if the following series converges or diverges. **Circle your final answer choice**. Fully justify your answer including using proper notation and showing mechanics of the comparison test.

$$\sum_{n=3}^{\infty} \frac{\ln(n)}{n + \ln(n)}$$

Circle one: **Converges** $\boxed{\textbf{Diverges}}$

Solution: We first note that $\ln n < n$ for all n, and $\ln n > 1$ for all $n \geq 3$. Therefore, we have

$$\frac{\ln n}{n + \ln n} > \frac{1}{n + \ln n} > \frac{1}{n + n} = \frac{1}{2n}, \text{ for } n \geq 3.$$

Next, by the p-test ($p = 1$), we know that $\sum_{n=3}^{\infty} \frac{1}{2n}$ diverges. Therefore, by the (direct) comparison test (along with the inequality above, and the fact that the terms of the original series are positive), we have that $\sum_{n=3}^{\infty} \frac{\ln(n)}{n + \ln(n)}$ must also diverge.