5. [8 points] Use the **comparison test** for series to determine if the following series converges or diverges. **Circle your final answer choice.** Fully justify your answer including using proper notation and showing mechanics of the comparison test.

\[ \sum_{n=3}^{\infty} \frac{\ln(n)}{n + \ln(n)} \]

**Circle one:**

- [ ] Converges
- Diverges

**Solution:** We first note that \( \ln n < n \) for all \( n \), and \( \ln n > 1 \) for all \( n \geq 3 \). Therefore, we have

\[
\frac{\ln n}{n + \ln n} > \frac{1}{n + \ln n} > \frac{1}{n + n} = \frac{1}{2n}, \text{ for } n \geq 3.
\]

Next, by the \( p \)-test \( (p = 1) \), we know that \( \sum_{n=3}^{\infty} \frac{1}{2n} \) diverges. Therefore, by the (direct) comparison test (along with the inequality above, and the fact that the terms of the original series are positive), we have that \( \sum_{n=3}^{\infty} \frac{\ln(n)}{n + \ln(n)} \) must also diverge.