6. [8 points] Determine whether the following series converges or diverges. If it converges, determine if it is absolute or conditional convergence. Circle your final answer choice. Fully justify your answer including using proper notation and showing mechanics of any tests you use.

\[ \sum_{n=3}^{\infty} \frac{(-1)^n}{n^2 - 2} \]

Circle one: Absolutely Converges  Conditionally Converges  Diverges

**Solution:** We first investigate absolute convergence, i.e. the convergence/divergence of the absolute value series

\[ \sum_{n=3}^{\infty} \frac{|(-1)^n|}{n^2 - 2} = \sum_{n=3}^{\infty} \frac{1}{n^2 - 2}. \]

Note that \( n^2 - 2 \geq \frac{1}{2} n^2 \), for \( n \geq 3 \). Therefore,

\[ \frac{1}{n^2 - 2} \leq \frac{2}{n^2}, \text{ for } n \geq 3. \]

Now, by the \( p \)-test \( (p = 2) \), we know that \( \sum_{n=3}^{\infty} \frac{2}{n^2} \) converges. Therefore, by the (direct) comparison test (along with the inequality above, and the fact that \( n^2 - 2 \) is positive), we have that \( \sum_{n=3}^{\infty} \frac{1}{n^2 - 2} \) must also converge. So, by the Absolute Convergence Test, since \( \sum_{n=3}^{\infty} \frac{1}{n^2 - 2} \) converges, we have that \( \sum_{n=3}^{\infty} \frac{(-1)^n}{n^2 - 2} \) (absolutely) converges.