6. [8 points] Determine whether the following series converges or diverges. If it converges, determine if it is absolute or conditional convergence. **Circle your final answer choice.** Fully justify your answer including using proper notation and showing mechanics of any tests you use.

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{n^2-2}$$

Circle one: **Absolutely Converges** **Conditionally Converges** **Diverges**

Solution: We first investigate absolute convergence, i.e. the convergence/divergence of the absolute value series $\sum_{n=3}^{\infty} \left|\frac{(-1)^n}{n^2-2}\right| = \sum_{n=3}^{\infty} \frac{1}{n^2-2}$.

Note that $n^2 - 2 \geq \frac{1}{2}n^2$, for $n \geq 3$. Therefore,

$$\frac{1}{n^2-2} \leq \frac{2}{n^2}, \text{ for } n \geq 3.$$

Now, by the p-test ($p = 2$), we know that $\sum_{n=3}^{\infty} \frac{2}{n^2}$ converges. Therefore, by the (direct) comparison test (along with the inequality above, and the fact that n^2-2 is positive), we have that $\sum_{n=3}^{\infty} \frac{1}{n^2-2}$ must also converge. So, by the Absolute Convergence Test, since $\sum_{n=3}^{\infty} \frac{1}{n^2-2}$ converges, we have that $\sum_{n=3}^{\infty} \frac{(-1)^n}{n^2-2}$ (absolutely) converges.