8. [12 points] The parts of this problem are unrelated to each other.
   a. [5 points] Compute the following limit. Fully justify your answer including using proper notation.
   \[
   \lim_{x \to \infty} 2x \ln \left(1 + \frac{5}{x}\right)
   \]

   **Solution:** We note that \( \lim_{x \to \infty} 2x = \infty \), and \( \lim_{x \to \infty} \ln \left(1 + \frac{5}{x}\right) = 0 \). So, the limit is in the “\( \infty \cdot 0 \)” form, allowing us to attempt to use L’Hôpital’s rule. So, we first express the limit as a fraction, and proceed as shown below.

   \[
   \lim_{x \to \infty} 2x \ln \left(1 + \frac{5}{x}\right) = 2 \lim_{x \to \infty} \ln \left(1 + \frac{5}{x}\right) \cdot \frac{1}{x} \\
   \text{L’H} \Rightarrow 2 \lim_{x \to \infty} \frac{\frac{1}{1+\frac{5}{x}} \cdot -\frac{5}{x^2}}{-\frac{1}{x^2}} \\
   = 2 \lim_{x \to \infty} \frac{1}{1 + \frac{5}{x}} \cdot 5 \\
   = 10.
   \]

   **Answer:** \( \lim_{x \to \infty} 2x \ln \left(1 + \frac{5}{x}\right) = 10 \)

   b. [7 points] Compute the value of the following improper integral if it converges. If it does not converge, use a direct computation of the integral to show its divergence. **Circle your final answer choice.** Show your full computation, and use proper notation.

   \[
   \int_0^3 \frac{1}{(x-3)^2} \, dx
   \]

   **Circle one:**  
   
   **Converges to:**  
   
   **Diverges**

   **Solution:** We note that \( \frac{1}{(x-3)^2} \) has a vertical asymptote at \( x = 3 \). Therefore, we express the given (improper) integral as follows, and proceed with direct computation.

   \[
   \int_0^3 \frac{1}{(x-3)^2} \, dx = \lim_{b \to 3^-} \int_0^b \frac{1}{(x-3)^2} \, dx \\
   = \lim_{b \to 3^-} \left[ -\frac{1}{x-3} \right]_0^b \\
   = \lim_{b \to 3^-} \left( -\frac{1}{b-3} - \frac{1}{3} \right) \\
   = +\infty \text{ (or DNE)}.
   \]

   Therefore, by direct computation, \( \int_0^3 \frac{1}{(x-3)^2} \, dx \) diverges.