

6. [10 points] Liban is writing songs using a new style of music which he calls “new-age jazz.” The longer that he spends writing a particular song, the better it turns out.

Let $Q(t)$ be the **cumulative distribution function** (cdf) for t , the number of days that it takes for Liban to write a particular song. The formula for $Q(t)$ is shown to the right, where $c > 0$ is a constant.

$$Q(t) = \begin{cases} 0 & t < 0, \\ \frac{c}{4}t^2 & 0 \leq t \leq 2, \\ 2c - ce^{2-t} & t > 2. \end{cases}$$

You do not need to show your work in this problem, but partial credit may be given for work shown.

- a. [3 points] Write a piecewise-defined formula for $q(t)$, the **probability density function** (pdf) corresponding to $Q(t)$. Your answer may involve c , but it should not involve the letter Q .

$$q(t) = \begin{cases} \text{_____} & t < 0, \\ \text{_____} & 0 \leq t \leq 2, \\ \text{_____} & t > 2. \end{cases}$$

- b. [3 points] Write an expression involving one or more integrals that represents the **mean** number of days that it takes for Liban to write a particular song. Your answer may involve c , but it should not involve the letters Q or q . **Do not evaluate your integral(s).**

Answer: _____

- c. [2 points] Use the fact that $Q(t)$ is a cumulative distribution function to find the value of c .

Answer: $c =$ _____

- d. [2 points] Circle the **one** correct answer below that completes the following sentence:

“The quantity $Q(5)$ represents...

- (i) ...the probability that it takes exactly 5 days for Liban to write a song.”
 (ii) ...the probability that it takes more than 5 days for Liban to write a song.”
 (iii) ...the probability that it takes 5 days or less for Liban to write a song.”
 (iv) ...the approximate probability that it takes between 4.5 and 5.5 days for Liban to write a song.”
 (v) NONE OF THESE