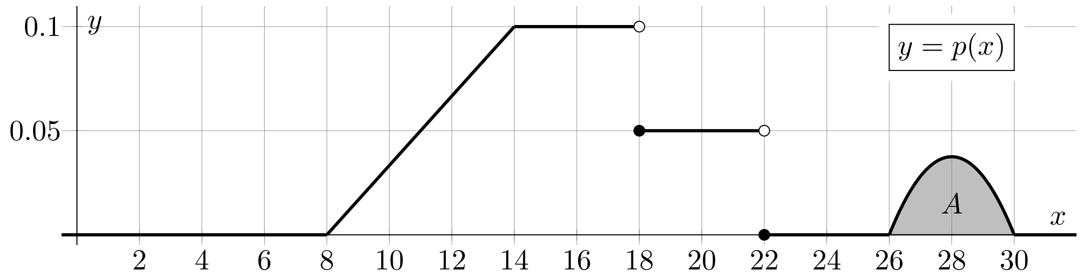
1. [9 points] Every Saturday during the summer, Dominic rides his bicycle in a national park. The distance he travels on his bicycle each Saturday varies.
Let $p(x)$ be the **probability density function** (pdf) for x, the distance (in miles) that Dominic bicycles on a Saturday. The graph of $p(x)$, shown below, has the following properties:

- $p(x)$ is piecewise linear for $x \leq 26$.
- $p(x)$ is nonzero only for $8 < x < 22$ and $26 < x < 30$.
- The area of the shaded region is A.



For each part of this problem, your answer should not involve the letter A. You do not need to show your work in this problem, but partial credit may be awarded for work shown clearly.

a. [1 point] Find the **minimum** distance that Dominic bicycles on a Saturday.

Solution: Since $p(x) = 0$ for $x < 8$, there is a 0% chance that Dominic bicycles less than 8 miles. Since $p(x) > 0$ for x slightly larger than 8, there is a nonzero chance that Dominic bicycles slightly more than 8 miles. Therefore the minimum distance he travels is 8 miles.

Answer: 8 miles

b. [2 points] Find the **median** distance that Dominic bicycles on a Saturday.

Solution: We must find the value of x for which $\int_{-\infty}^{x} p(t)\,dt = 0.5$. This is given by $x = 16$ because, by adding areas of triangles and rectangles,

$$\int_{-\infty}^{16} p(t)\,dt = \int_{8}^{14} p(t)\,dt + \int_{14}^{16} p(t)\,dt = \frac{1}{2}(6)(0.1) + 2(0.1) = 0.3 + 0.2 = 0.5.$$

Answer: 16 miles

c. [2 points] Use the fact that $p(x)$ is a probability density function to find the value of A.

Solution: Since $p(x)$ is a probability density function, we must have $\int_{-\infty}^{\infty} p(t)\,dt = 1$. Adding up areas, we have

$$\int_{8}^{22} p(t)\,dt = \frac{1}{2}(6)(0.1) + 4(0.1) + 4(0.05) = 0.3 + 0.4 + 0.2 = 0.9.$$

Then $0.9 + A = 1$ implies $A = 0.1$.

Answer: $A =$ 0.1

d. [2 points] Calculate the probability that Dominic bicycles farther than 18 miles on a Saturday.

Solution: There are two possible ways to solve this problem:

Solution 1 (Using A)**:** Computing the probability directly, this is given by

$$\int_{18}^{\infty} p(t)\,dt = \int_{18}^{22} p(t)\,dt + A = 4(0.05) + A = 0.2 + A.$$

In part (c), we found that $A = 0.1$, so our answer is $0.2 + 0.1 = 0.3$.

Solution 2 (Without A)**:** The probability that Dominic bicycles for 18 miles or less is given by

$$\int_{-\infty}^{18} p(t)\,dt = \frac{1}{2}(6)(0.1) + 4(0.1) = 0.3 + 0.4 = 0.7.$$

Then the probability that Dominic bicycles for more than 18 miles is given by $1 - 0.7 = 0.3$.

Answer: 0.3 **or** 30%

e. [2 points] Complete the sentence below to write a practical interpretation of the equation $p(28) = 0.0375$:

The probability that Dominic bicycles between 27 and 29 miles on a Saturday is...

Solution: ...approximately $2(0.0375) = 0.075$, or 7.5%.