3. [13 points] Consider the following sequences, each defined for $n \geq 1$:

$$a_n = \frac{\cos(\pi n)}{n} \qquad b_n = -\left(\frac{100}{99}\right)^n \qquad c_n = \sum_{k=0}^{n} \frac{1}{3^k}$$

a. [9 points] For each of the sequences above, determine whether the sequence is bounded, whether it is monotone, and whether it is convergent. No justification is required.

(i) The sequence a_n is...

Circle one: $\boxed{\textbf{Bounded}}$ **Unbounded**

Circle one: **Monotone** $\boxed{\textbf{Not Monotone}}$

Circle one: $\boxed{\textbf{Convergent}}$ **Divergent**

(ii) The sequence b_n is...

Circle one: **Bounded** $\boxed{\textbf{Unbounded}}$

Circle one: $\boxed{\textbf{Monotone}}$ **Not Monotone**

Circle one: **Convergent** $\boxed{\textbf{Divergent}}$

(iii) The sequence c_n is...

Circle one: $\boxed{\textbf{Bounded}}$ **Unbounded**

Circle one: $\boxed{\textbf{Monotone}}$ **Not Monotone**

Circle one: $\boxed{\textbf{Convergent}}$ **Divergent**

b. [4 points] Determine whether the following series is convergent or divergent. **Fully justify** your answer including using **proper notation** and **showing mechanics** of any tests you use. Circle your final answer choice.

$$\sum_{n=0}^{\infty} c_n$$

Circle one: **Convergent** $\boxed{\textbf{Divergent}}$

Solution: Note that

$$\lim_{n\to\infty} c_n = \lim_{n\to\infty} \sum_{k=0}^{n} \frac{1}{3^k} = \sum_{k=0}^{\infty} \frac{1}{3^k} = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k = \frac{1}{1-\frac{1}{3}} = \frac{3}{2} \neq 0.$$

Therefore, by the ***n*th term test for divergence,** the series $\sum_{n=0}^{\infty} c_n$ is **divergent.**