11. [12 points] For the following questions, determine if the statement is ALWAYS true, SOMETIMES true, or NEVER true, and circle the corresponding answer. Justification is not required.

a. [2 points] Let $F(x)$ be a cumulative distribution function (cdf) that is continuous for all x. Then, the series $\sum_{n=1}^{\infty}(-1)^n(1-F(n))$ converges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

b. [2 points] Let $f(x)$ be a non-negative, continuous function for all $x \geq 1$, and suppose that $f(x)$ is decreasing. Then, the integral $\int_1^{\infty} \frac{f(x)}{x}\,\mathrm{d}x$ converges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

c. [2 points] Let $g(x)$ be a continuous function and suppose that for all n, $s_n = g(n)$. If $\int_1^{\infty} g(x)\,\mathrm{d}x$ diverges, then $\sum_{n=1}^{\infty} s_n$ also diverges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

d. [2 points] Let $h(x)$ be a non-negative, continuous function for all x, and suppose that $h(x)$ is decreasing. Let $a_n = h(n)$ for all n. If $\int_1^{\infty} xh(x)\,\mathrm{d}x$ converges, then the series $\sum_{n=1}^{\infty} a_n$ also converges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

e. [2 points] Let $b_n \geq 0$ and $c_n \geq 0$ for all n. Suppose that the series $\sum_{n=1}^{\infty} b_n$ converges and that the sequence c_n also converges. Then, the series $\sum_{n=1}^{\infty} b_n 2^{c_n}$ diverges.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

f. [2 points] Suppose that d_n is a monotonic decreasing sequence of positive numbers that converges to 0. Furthermore, assume that $\lim_{n\to\infty} \frac{d_n}{1/n^2} = 5$. Then, the series $\sum_{n=1}^{\infty}(-1)^n d_n$ is conditionally convergent.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**