6. [12 points] Kalani and his friends are in Honolulu, Hawaii, enjoying their spring break. One day, they decide to try a new virtual surfing game. Let $P(t)$ denote the cumulative distribution function (cdf) representing the probability that a first-time player takes t minutes or less to complete the game. According to the developer's data, the formula of $P(t)$ is given by

$$P(t) = \begin{cases} 0, & t \leq 0, \\ \frac{a}{9}t^2, & 0 < t \leq 3, \\ \frac{b}{2} - ae^{3-t}, & t > 3. \end{cases}$$

where $a > 0$ and $b > 0$ are constants.

a. [4 points] The function $P(t)$ is a **continuous** cumulative distribution function (cdf). Find the values of a and b.

Answer: $a =$ _____ and $b =$ _____

b. [2 points] Write an expression for the probability that a first-time player takes at least 1 minute and at most 7 minutes to complete the game. Your answer may include the letters a and b, but it should not involve the letter P. Your answer should **not** include integrals.

Answer: _____

c. [3 points] Write a piecewise-defined formula for $p(t)$, the probability density function (pdf) corresponding to $P(t)$. Your answer may include the letters a and b, but it should not include the letter P.

$$p(t) = \begin{cases} \_\_\_\_\_, & t \leq 0 \\ \_\_\_\_\_, & 0 < t \leq 3, \\ \_\_\_\_\_, & t > 3. \end{cases}$$

d. [3 points] Write an expression involving one or more integrals that represents the mean time (in minutes) it takes for a first-time player to complete the virtual surfing game. Your answer may include the letters a and b, but it should not involve the letters P or p. **Do not evaluate your integral(s).**

Answer: _____