2. [4 points] Compute the following limit. Fully justify your answer including using **proper limit notation**.

$$\lim_{x\to 1}\frac{\sin(\ln(x))}{x^2-1}$$

Solution: Since $\ln 1 = 0$, the limit is in the indeterminate form "$\frac{0}{0}$". We have

$$\lim_{x\to 1}\frac{\sin(\ln(x))}{x^2-1} \overset{\text{L'H}}{=} \lim_{x\to 1}\frac{\frac{\cos(\ln(x))}{x}}{2x} = \frac{\cos(\ln(1))}{2(1)^2} = \frac{\cos(0)}{2} = \frac{1}{2}.$$

Answer: $\lim_{x\to 1}\frac{\sin(\ln(x))}{x^2-1} =$ $\frac{1}{2}$

3. [7 points] **Compute** the value of the following improper integral if it converges.
If it does not converge, use a **direct computation** of the integral to show its divergence.
Be sure to show your full computation, and be sure to use **proper notation**.

$$\int_0^\infty \frac{x}{e^{3x^2}}\,\mathrm{d}x$$

Solution: By definition,

$$\int_0^\infty \frac{x}{e^{3x^2}}\,\mathrm{d}x = \lim_{b\to\infty}\int_0^b \frac{x}{e^{3x^2}}\,\mathrm{d}x.$$

Set $u = 3x^2$. Then $\mathrm{d}u = 6x\,\mathrm{d}x$. Using the method of substitution,

$$\int \frac{x}{e^{3x^2}}\,\mathrm{d}x = \frac{1}{6}\int e^{-u}\,\mathrm{d}u = -\frac{e^{-u}}{6} + C = -\frac{e^{-3x^2}}{6} + C.$$

Therefore,

$$\begin{aligned}\int_0^\infty \frac{x}{e^{3x^2}}\,\mathrm{d}x &= \lim_{b\to\infty}\int_0^b \frac{x}{e^{3x^2}}\,\mathrm{d}x \\ &= \lim_{b\to\infty}\left[-\frac{e^{-3x^2}}{6}\Bigg|_0^b\right] \\ &= \lim_{b\to\infty}\left[-\frac{e^{-3b^2}}{6} + \frac{1}{6}\right] \\ &= 0 + \frac{1}{6} \\ &= \frac{1}{6}.\end{aligned}$$

Circle one: **Diverges** **Converges to** $\frac{1}{6}$