4. [7 points] Determine if the following series converges or diverges using the Limit Comparison Test, and circle the corresponding word. Fully justify your answer including using proper notation and showing mechanics of any tests you use.

$$\sum_{n=1}^{\infty} \frac{2n-1}{\sqrt{9n^4 - 10n + 3}}$$

Circle one:

Diverges

Justification (using the Limit Comparison Test):

Solution: We compare the given series with the series $\sum_{n=1}^{\infty} \frac{1}{n}$. We have

Converges

$$\lim_{n \to \infty} \frac{\frac{2n-1}{\sqrt{9n^4 - 10n + 3}}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{(2n-1)n}{\sqrt{9n^4 - 10n + 3}} = \lim_{n \to \infty} \frac{2n^2}{3n^2} = \frac{2}{3} > 0.$$

The series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by the *p*-test (p = 1). Therefore, $\sum_{n=1}^{\infty} \frac{2n-1}{\sqrt{9n^4-10n+3}}$ diverges by the Limit Comparison Test (LCT).