

7. [7 points] Determine whether the following improper integral converges or diverges and circle the corresponding word. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use. You do not need to calculate the value of the integral if it converges.

$$\int_1^{\infty} \frac{\cos\left(\frac{1}{t}\right)}{t^{2/5}} dt$$

Circle one:

Converges

**Diverges**

Justification:

*Solution:* We use the Direct Comparison Test (DCT) to determine the convergence or divergence of the given integral. Observe that  $\cos\left(\frac{1}{t}\right) \geq \cos(1)$  for  $t \geq 1$ . Therefore,

$$\frac{\cos\left(\frac{1}{t}\right)}{t^{2/5}} \geq \frac{\cos(1)}{t^{2/5}}, \quad t \geq 1.$$

The integral  $\int_1^{\infty} \frac{\cos(1)}{t^{2/5}} dt$  diverges by the  $p$ -test ( $p = 2/5$ ). Therefore,  $\int_1^{\infty} \frac{\cos\left(\frac{1}{t}\right)}{t^{2/5}} dt$  diverges by the Direct Comparison Test (DCT).