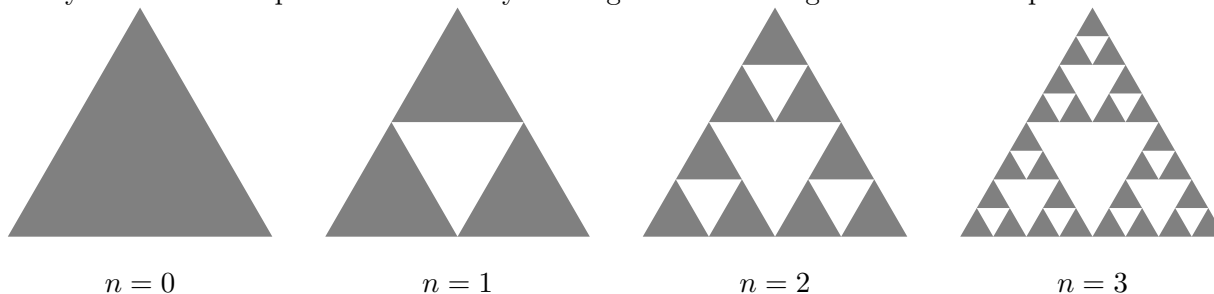


10. [9 points] Molly constructs a pattern using paper in the shape of an equilateral triangle of side length 1.

- **Step 1:** Molly divides the triangle into 4 congruent triangles, and removes the triangle in the middle.
- **Step 2:** For each of the remaining triangles, Molly repeats the previous step.

Molly continues this process indefinitely. A diagram illustrating the first few steps is shown below.



The area of an equilateral triangle with side length  $a$  is given by  $\frac{\sqrt{3}}{4}a^2$ .

- a. [4 points] For  $n \geq 1$ , let  $A_n$  be the **total** area of all the triangles Molly has **removed** after  $n$  steps. For example,  $A_1 = \frac{\sqrt{3}}{4} \left(\frac{1}{2}\right)^2$ . Find  $A_2$  and  $A_3$ . You do not need to simplify your expressions.

**Answer:**  $A_2 =$  \_\_\_\_\_

**Answer:**  $A_3 =$  \_\_\_\_\_

- b. [4 points] Find a **closed-form** expression for  $A_n$ . Closed form means your answer should not include ellipses or sigma notation, and should NOT be recursive.

**Answer:**  $A_n =$  \_\_\_\_\_

- c. [1 point] Find the limit  $\lim_{n \rightarrow \infty} A_n$ .

**Answer:**  $\lim_{n \rightarrow \infty} A_n =$  \_\_\_\_\_