

11. [12 points]

- a. [6 points] For the following questions, determine if the statement is ALWAYS true, SOMETIMES true, or NEVER true, and circle the corresponding answer. Justification is not required.

(i) Suppose that the sequence a_n is defined by the recurrence relation $a_{n+1} = -\frac{1}{a_n}$ for all $n \geq 1$, where $a_1 > 0$. Then the sequence a_n is monotone.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

(ii) Suppose that b_n is a sequence and that $S_n = \sum_{k=1}^n b_k$ is the corresponding sequence of partial sums. Suppose that $\sum_{k=1}^{\infty} b_k$ converges. Then the sequence S_n is bounded.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

(iii) Suppose that c is a positive constant. Then the geometric series $1 + 2c + 4c^2 + 8c^3 + \dots$ converges to $\frac{1}{1-2c}$.

Circle one: **ALWAYS** **SOMETIMES** **NEVER**

- b. [6 points] For each of the following sequences, defined for $n \geq 1$, decide whether the sequence is monotone increasing, monotone decreasing, or not monotone, and whether it is bounded or unbounded. Circle your answers. No justification is required.

(i) $r_n = e^{n+1} - e^n$

Circle **all** which apply:

Monotone Increasing Monotone Decreasing Not Monotone
Bounded Unbounded

(ii) $s_n = \int_0^{2^n} e^{-x^2} dx$

Circle **all** which apply:

Monotone Increasing Monotone Decreasing Not Monotone
Bounded Unbounded

(iii) $t_n = \sum_{k=1}^n \frac{(-1)^k}{k}$

Circle **all** which apply:

Monotone Increasing Monotone Decreasing Not Monotone
Bounded Unbounded