2. [4 points] Compute the following limit. Fully justify your answer and use **proper limit notation**.

$$\lim_{x\to 0} \frac{\sin^2(7x)}{\cos(7x)-1}$$

Solution: We use L'Hospital's Rule:

$$\begin{aligned}\lim_{x\to 0} \frac{\sin^2(7x)}{\cos(7x)-1} &\overset{\text{LH}}{=} \lim_{x\to 0} \frac{14\sin(7x)\cos(7x)}{-7\sin(7x)} \\ &= \lim_{x\to 0} -2\cos(7x) \\ &= -2.\end{aligned}$$

Answer: $\lim_{x\to 0} \frac{\sin^2(7x)}{\cos(7x)-1} =$ -2

3. [7 points] **Compute** the value of the following improper integral if it converges. If it does not converge, use a **direct computation** of the integral to show its divergence. Be sure to show your full computation, and be sure to use **proper notation**.

$$\int_2^\infty \frac{e^{-3/x}}{x^2}\,dx$$

Solution: By definition,

$$\int_2^\infty \frac{e^{-3/x}}{x^2}\,dx = \lim_{b\to\infty} \int_2^b \frac{e^{-3/x}}{x^2}\,dx.$$

Set $u = \frac{3}{x}$. Then $\mathrm{d}u = -\frac{3}{x^2}\,\mathrm{d}x$.
Therefore,

$$\begin{aligned}\int_2^\infty \frac{e^{-3/x}}{x^2}\,dx &= \lim_{b\to\infty} \int_2^b \frac{e^{-3/x}}{x^2}\,dx. \\ &= \lim_{b\to\infty} \int_{3/2}^{3/b} -\frac{1}{3}e^{-u}\,du. \\ &= \lim_{b\to\infty} \frac{1}{3}e^{-u}\Big|_{3/2}^{3/b} \\ &= \lim_{b\to\infty} \frac{1}{3}e^{-3/b} - \frac{1}{3}e^{-3/2} \\ &= \frac{1}{3} - \frac{1}{3}e^{-3/2}\end{aligned}$$

Circle one: **Diverges** $\boxed{\textbf{Converges to } \frac{1}{3}\left(1-e^{-3/2}\right)}$