

4. [7 points] Determine whether the following improper integral converges or diverges and circle the corresponding word. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use. You do not need to calculate the value of the integral if it converges.

$$\int_0^1 \frac{7}{x^{1/3} + x^{4/3}} \, dx$$

Circle one:

Converges

Diverges

Justification:

Solution: On the interval $0 \leq x \leq 1$, we have $\frac{7}{x^{1/3} + x^{4/3}} \leq \frac{7}{x^{1/3}}$, and $\int_0^1 \frac{7}{x^{1/3}} \, dx$ converges by the p -test with $p = \frac{1}{3}$. Therefore, by the (Direct) Comparison Test, $\int_0^1 \frac{7}{x^{1/3} + x^{4/3}} \, dx$ converges.