6. [7 points] Determine if the following series converges or diverges using the **Limit Comparison Test**, and circle the corresponding word. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use.

$$\sum_{n=1}^{\infty} \frac{\sqrt{2n^3 - n^2 + 1}}{5n^2 + n - 4}$$

Circle one: **Converges** $\boxed{\textbf{Diverges}}$

*Justification (using the **Limit Comparison Test**):*

Solution: We aim to compare with the series $\sum \frac{1}{n^{1/2}}$. We see that

$$\begin{aligned}\lim_{n\to\infty} \frac{\frac{\sqrt{2n^3-n^2+1}}{5n^2+n-4}}{\frac{1}{n^{1/2}}} &= \lim_{n\to\infty} \frac{n^{1/2}\sqrt{2n^3 - n^2 + 1}}{5n^2 + n - 4} \\ &= \lim_{n\to\infty} \frac{\sqrt{2}n^2}{5n^2} \\ &= \frac{\sqrt{2}}{5},\end{aligned}$$

and so the Limit Comparison Test does apply.

By the p-test for series (with $p = \frac{1}{2}$), $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges. Therefore, by the Limit Comparison Test,

$$\sum_{n=1}^{\infty} \frac{\sqrt{2n^3 - n^2 + 1}}{5n^2 + n - 4}$$

also diverges.