**7**. [10 points] Determine if the following series absolutely converges, conditionally converges, or diverges, and circle the corresponding word. **Fully justify** your answer including using **proper notation** and showing mechanics of any tests you use.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sin\left(\frac{1}{n}\right) + \sqrt{n^2+1}}$$

*Hint: the function* $\sin(\frac{1}{x}) + \sqrt{x^2+1}$ *is a monotone increasing function for* $x \geq 2$.

*Circle one:* **Converges Absolutely** $\boxed{\textbf{Converges Conditionally}}$ **Diverges**

*Justification:*

*Solution:* Note that the series is alternating. Let $a_n = \dfrac{1}{\sin\left(\frac{1}{n}\right) + \sqrt{n^2+1}}$. Then for all $n$, $0 < a_{n+1} < a_n$ (using the hint), and we also have $\lim\limits_{n\to\infty} a_n = 0$. Therefore, by the Alternating Series Test, $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sin\left(\frac{1}{n}\right) + \sqrt{n^2+1}}$ converges.

Now consider the series $\displaystyle\sum_{n=2}^{\infty} \left| \frac{(-1)^n}{\sin\left(\frac{1}{n}\right) + \sqrt{n^2+1}} \right| = \sum_{n=2}^{\infty} \frac{1}{\sin\left(\frac{1}{n}\right) + \sqrt{n^2+1}}$.

Note that for $n \geq 2$, we have $\dfrac{1}{\sin\left(\frac{1}{n}\right) + \sqrt{n^2+1}} \geq \dfrac{1}{n + \sqrt{2n^2}} = \dfrac{1}{(1+\sqrt{2})n}$.

By the $p$-test, with $p = 1$, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(1+\sqrt{2})n}$ diverges, and so by the (Direct) Comparison Test, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sin\left(\frac{1}{n}\right) + \sqrt{n^2+1}}$ diverges.

Therefore, $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\sin\left(\frac{1}{n}\right) + \sqrt{n^2+1}}$ converges conditionally.