**9.** ( 15 points) Frodo Baggins of the Shire is given the task of taking the ring of power from the elven kingdom of Rivendell to Mount Doom, 100 km away, to destroy it. The ring's weight $w$ (in kg) grows at a rate of one one-hundredth (1/100) of Frodo's distance $x$ (in km) from Rivendell as Frodo proceeds on his journey. Frodo can travel toward Mount Doom at the rate of 2.5 km per hour except that the weight of the ring of power slows his rate of travel (in km/hr) to Mount Doom by one-twentieth (1/20) of the weight of the ring (in kg). Suppose that the ring weighs .001 kg (1 gram) when he begins his journey from Rivendell.

**(a)** Write a pair of differential equations for the functions $w(t)$ and $x(t)$ that give the weight of the ring and Frodo's distance from Rivendell on the road to Mount Doom $t$ hours after leaving Rivendell. What are the initial conditions at time $t = 0$?

**(b)** What differential equation models the relationship between the weight of the ring and the distance from Rivendell?

*Problem continued on next page.*