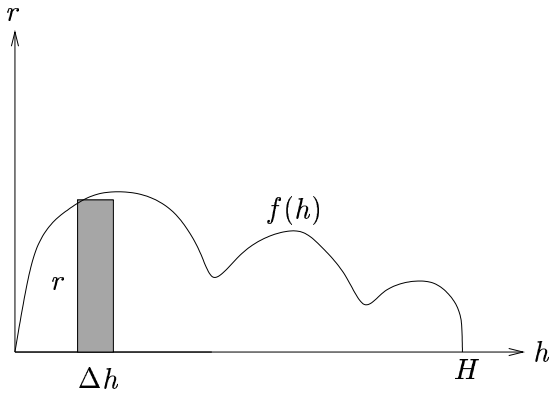
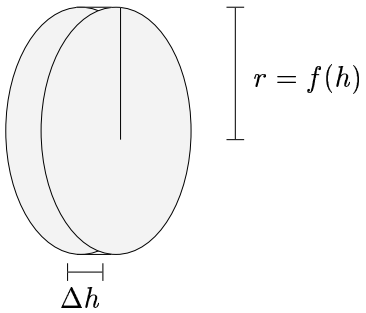
3. (10 points) The cross-sections of a snowman are given by circles of radius $r = f(h)$ where h is the height measured from the ground and $f(h)$ has graph given in the figure. Both r and h are measured in inches.



(a) Draw and label a typical, thin, cross-section of the snowman. What is the volume of the cross-section (in terms of the function $f(h)$)?



The volume of a typical cross-section can now be read directly from the picture to be

$$V_{\text{slice}} = \pi\, r^2\, \Delta h = \pi\, [f(h)]^2\, \Delta h.$$

(b) Write an integral in terms of $f(h)$ whose value is the total amount of snow used in making the snowman.

To find the total amount of snow used to make the snowman, we add up the volumes of all of the slices forming a Riemann sum, and then let $\Delta h \to 0$. The resulting integral is

$$\int_0^H \pi\, [f(h)]^2\, dh.$$