7. (10 points) During the holiday season there are two main groups of people at the mall (excluding the store employees). These are the shoppers and the volunteers ringing bells to collect money for charities. The numbers of each vary over time. If we let $B(t)$ be the number of bell ringers at time t and $S(t)$ be the number of shoppers at time t, and assume these are modeled by a predator-prey system of differential equations, then the differential equations describing their numbers are

$$\begin{aligned} \frac{dB}{dt} &= -1,000\,B + 2\,BS \\ \frac{dS}{dt} &= 66\,S - 11\,BS. \end{aligned}$$

(a) Given this model, which is the "predator" and which is the "prey"? Make sure you justify your answer by explaining how this is reflected in the given equations.

The bellringers, B, are the predators since their population in increased by the interaction between the two species, (i.e. the $+2\,BS$ term), while the population of shoppers, S is decreased by the interaction term ($-11\,BS$).

(b) What are the equilibrium points of this system? Describe what the equilibrium points mean in terms of this problem.

The equilibrium points are where the poputlation of shoppers and bell ringers does not change with time, or $dB/dt = dS/dt = 0$. That is,

$$-1000B + 2\,BS = B(-1000B + 2S) = 0$$
$$66S - 11BS = S(66 - 11S) = 0$$

which has two solutions, namely $(B, S) = (0, 0)$ and $(B, S) = (6, 500)$. The first equilibrium solution represents the situation where there are no bell ringers or shoppers. The second means that, according to this model, when there are 6 bell ringers and 500 shoppers, the number of bell ringers and shoppers will remain constant.