10. (8 points) We shall investigate a well-known physical phenomenon, called the "Doppler Effect". When an electromagnetic signal (e.g. a ray of light) with frequency F_e is emitted from a source moving away with velocity $v > 0$ with respect to a receiver at rest, then the received frequency F_r is different from F_e. The relationship linking the emitted frequency F_e and the received frequency F_r is the Doppler Law:

$$F_r \;=\; \sqrt{\frac{1-v/c}{1+v/c}}\, F_e \;, \qquad \text{where } c \text{ is a constant, the speed of light.}$$

For this problem, you might find useful to know that the Taylor series for the function $\sqrt{\dfrac{1+x}{1-x}}$ near $x = 0$ is $1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{2} + \cdots$.

(a) On Earth, nearly all objects travel with velocities v much smaller than the speed of light c, i.e. the ratio v/c is very small. Use this fact to obtain the approximation to the Doppler Law for slow-moving emitters:

$$F_r \;\simeq\; \left(1 - \frac{v}{c}\right) F_e \,.$$

(b) The relationship in part **(a)** is *not* exact, and an error is made when it is used to approximate the Doppler Law. Find an expression for the "error", in terms of v, c and F_e. Is the approximation accurate within 1% of F_e if the velocity is at most 20% of the speed of light c? *Explain.*