11. (13 points) In normal conditions, the thyroid hormone (Hormone T), produced in the thyroid gland, and the thyroid successor hormone (Hormone S), produced in the pituitary gland, form a so-called “auto-regulated feedback process”. The amount of one in the bloodstream influences the production of the other, and vice-versa. The simple system given below models this process, where $x$ is the amount of Hormone T (in standard units), and $y$ is the amount of Hormone S (in standard units), present in the bloodstream at time $t$ hours.

\[
\frac{dx}{dt} = 3 - y, \quad \frac{dy}{dt} = x - 2.
\]

(a) Find all equilibrium solutions (if any) of the system.

(b) Suppose that at $t = 0$, the amount of Hormone T in the blood was 1.0 and the amount of Hormone S was 3.5, both in standard units. Find the equation of the trajectory of the corresponding solution curve in the phase plane. Show your work.

Problem continued on next page.
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If the patient’s diet lacks iodine (e.g. from salt), the chemical agent responsible for detecting the presence of Hormone S in the blood is no longer active. The above model must be replaced by the new system:

\[
\frac{dx}{dt} = 3 - y + x, \quad \frac{dy}{dt} = x - 2 + \frac{y}{2}.
\]

The slope field for the differential equation that describes the trajectories of this system is shown on the figure below.

\[(c)\] Sketch on the figure the trajectory corresponding to the initial values in part \((b)\); that is, \(x(0) = 1.0\) and \(y(0) = 3.5\). \textit{You need not solve any differential equation.}

\[(d)\] In the context of this problem, briefly describe how the amounts of the hormones change from their initial values as time increases.