11. (13 points) In normal conditions, the thyroid hormone (Hormone T), produced in the thyroid gland, and the thyroid successor hormone (Hormone S), produced in the pituitary gland, form a so-called "auto-regulated feedback process". The amount of one in the bloodstream influences the production of the other, and vice-versa. The simple system given below models this process, where x is the amount of Hormone T (in standard units), and y is the amount of Hormone S (in standard units), present in the bloodstream at time t hours.

$$\frac{dx}{dt} = 3 - y\,, \qquad \frac{dy}{dt} = x - 2\,.$$

(a) Find all equilibrium solutions (if any) of the system.

(b) Suppose that at $t = 0$, the amount of Hormone T in the blood was 1.0 and the amount of Hormone S was 3.5, both in standard units. Find the equation of the trajectory of the corresponding solution curve in the phase plane. *Show your work.*

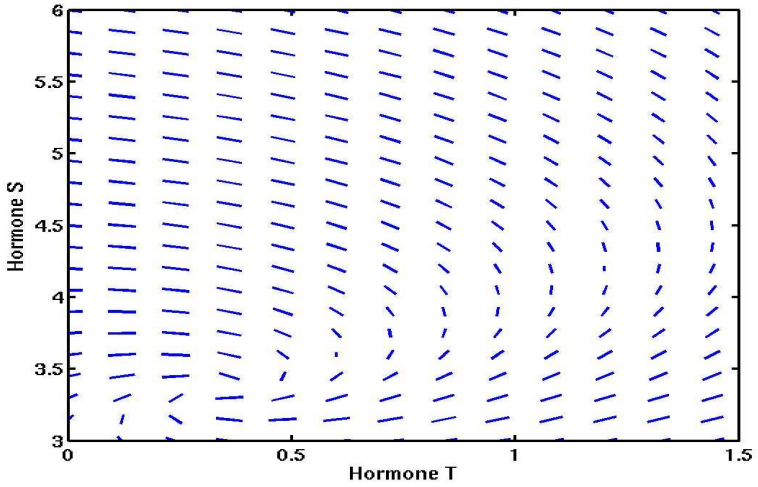
Problem continued on next page.

Problem continued from previous page.

If the patient's diet lacks iodine (e.g. from salt), the chemical agent responsible for detecting the presence of Hormone S in the blood is no longer active. The above model must be replaced by the new system:

$$\frac{dx}{dt} = 3 - y + x\,, \qquad \frac{dy}{dt} = x - 2 + \frac{y}{2}\,.$$

The slope field for the differential equation that describes the trajectories of this system is shown on the figure below.



(c) Sketch on the figure the trajectory corresponding to the initial values in part **(b)**; that is, $x(0) = 1.0$ and $y(0) = 3.5$. *You need not solve any differential equation.*

(d) In the context of this problem, briefly describe how the amounts of the hormones change from their initial values as time increases.