1. (16 points)

(a) If \( \int_0^1 f(x) \, dx = 2 \), then \( \int_0^2 f \left( \frac{x}{2} \right) \, dx = \frac{4}{2} \).

By substituting \( u = x/2 \), we obtain \( dx = 2 \, du \), and the interval of integration \( 0 \leq x \leq 2 \) changes to \( 0 \leq u \leq 1 \). Hence
\[
\int_0^2 f \left( \frac{x}{2} \right) \, dx = 2 \int_0^1 f(u) \, du = 2 \cdot 2 = 4 .
\]

(b) Does the infinite series \( \sum_{n=1}^{\infty} \frac{2^n}{n!} \) converge or diverge? Justify your answer.

We will use the Ratio Test with the general term \( a_n = \frac{2^n}{n!} \). Doing so yields:
\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{2^{n+1}}{2^n} \frac{n!}{(n+1)!} = 2 \lim_{n \to \infty} \frac{1}{n+1} = 0,
\]
which is smaller than 1. Therefore, the Ratio Test allows us to conclude that the given series converges.

Another solution consists in noting that since \( e^x = 1 + \sum_{n=1}^{\infty} \frac{x^n}{n!} \) holds true for all values of \( x \), the given series is actually equal to \( e^2 - 1 \), which is finite. Therefore, the series converges.

(c) Is the function \( te^{-2t} \) a solution of the differential equation \( \frac{dy}{dt} + 2y - e^{-2t} = 0 \)? Explain why or why not.

If \( y = te^{-2t} \), then \( \frac{dy}{dt} = e^{-2t} - 2te^{-2t} = e^{-2t} - 2y \).

Therefore, indeed, the given function does satisfy the differential equation, it is a solution.

(d) Suppose \( C(t) \) is the daily cost of heating your house, measured in dollars per day, where \( t = 0 \) corresponds to January 1, 2004. Give the meaning, in words, of each of the following quantities.

(i) \( \int_0^{60} C(t) \, dt \).

This integral represents the total cost in dollars of heating your house during the first 60 days of the year 2004, i.e. throughout January and February 2004.

(ii) \( \frac{1}{60} \int_0^{60} C(t) \, dt \).

This quantity represents the average cost in dollars per day of heating your house during the first 60 days of the year 2004, i.e. throughout January and February 2004.