1. (16 points)

(a) If $\int_0^1 f(x)\,dx = 2$, then $\int_0^2 f\left(\frac{x}{2}\right)\,dx =$ **4**.

By substituting $u = x/2$, we obtain $dx = 2\,du$, and the interval of integration $0 \le x \le 2$ changes to $0 \le u \le 1$. Hence

$$\int_0^2 f\left(\frac{x}{2}\right)\,dx \;=\; 2\int_0^1 f(u)\,du \;=\; 2\cdot 2 \;=\; \mathbf{4}\,.$$

(b) Does the infinite series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converge or diverge? *Justify your answer.*

We will use the Ratio Test with the general term $a_n = \frac{2^n}{n!}$. Doing so yields:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \;=\; \lim_{n\to\infty}\frac{2^{n+1}}{2^n}\,\frac{n!}{(n+1)!} \;=\; 2\lim_{n\to\infty}\frac{1}{n+1} \;=\; 0\,,$$

which is smaller than 1. Therefore, the Ratio Test allows us to conclude that the given series **converges**.

Another solution consists in noting that since $e^x = 1 + \sum_{n=1}^{\infty}\frac{x^n}{n!}$ holds true for all values of x, the given series is actually **equal to** $e^2 - 1$, *which is finite. Therefore, the series converges.*

(c) Is the function te^{-2t} a solution of the differential equation $\frac{dy}{dt} + 2y - e^{-2t} = 0$? *Explain why or why not.*

If $y = te^{-2t}$, then

$$\frac{dy}{dt} \;=\; e^{-2t} - 2\,te^{-2t} \;=\; e^{-2t} - 2y\,.$$

Therefore, indeed, the given function does satisfy the differential equation, **it is a solution**.

(d) Suppose $C(t)$ is the daily cost of heating your house, measured in dollars per day, where $t = 0$ corresponds to January 1, 2004. Give the meaning, in words, of each of the following quantities.

(i) $\int_0^{60} C(t)\,dt\,.$

This integral represents the **total cost in dollars** *of heating your house during the first 60 days of the year 2004, i.e. throughout January and February 2004.*

(ii) $\frac{1}{60}\int_0^{60} C(t)\,dt\,.$

This quantity represents the **average cost in dollars per day** *of heating your house during the first 60 days of the year 2004, i.e. throughout January and February 2004.*