

2. (6 points) Write a parametrization for each of the following curves in the xy -plane.

(a) The circle of radius 2, centered at the origin, traced clockwise, and starting from $(-2, 0)$ when $t = 0$.

One possible parametrization (there are many others) is:

$$x(t) = -2 \cos(t), \quad y(t) = 2 \sin(t), \quad 0 \leq t < 2\pi.$$

(b) The line passing through the points $(2, -1)$ and $(1, 3)$.

One possible parametrization (there are many others) is:

$$x(t) = t, \quad y(t) = -4t + 7.$$

3. (5 points)

(a) Briefly explain the difference between the indefinite integral $\int f(x) dx$ and the (proper) definite integral $\int_a^b f(x) dx$.

(See the text, §6.2, third paragraph on page 268.) “The definite integral is a number and the indefinite integral $\int f(x) dx$ is a family of functions.”

The indefinite integral is the family of all functions that are antiderivatives of f . Any two functions in the family differ by a constant. The definite (proper) integral $\int_a^b f(x) dx$ is a number. When $f(x) \geq 0$, it is equal to the area of the region above the x -axis and below the graph of the function $f(x)$ between $x = a$ and $x = b$.

(b) What is a Riemann sum and how is it related to one or more of the integrals in part (a)?

(See the text, §5.2, box on page 234.) A Riemann sum for f on an interval $[a, b]$ is a sum of the form

$$\sum_{i=1}^n f(c_i) \Delta x_i$$

where $a = x_0 < x_1 < \cdots < x_n = b$, and, for $i = 1, 2, \dots, n$, $\Delta x_i = x_i - x_{i-1}$, and $x_{i-1} \leq c_i \leq x_i$.

The definite integral $\int_a^b f(x) dx$ is defined to be the limit of the Riemann sums when the maximum length of the intervals $[x_{i-1}, x_i]$ tends to 0. Thus, a Riemann sum can be viewed as an approximation to the definite integral.

Geometrically, the Riemann sum may be viewed as the sum of areas of rectangles of height $f(c_i)$ and width $x_i - x_{i-1}$, a geometric approximation to the area of the region under the graph of f .