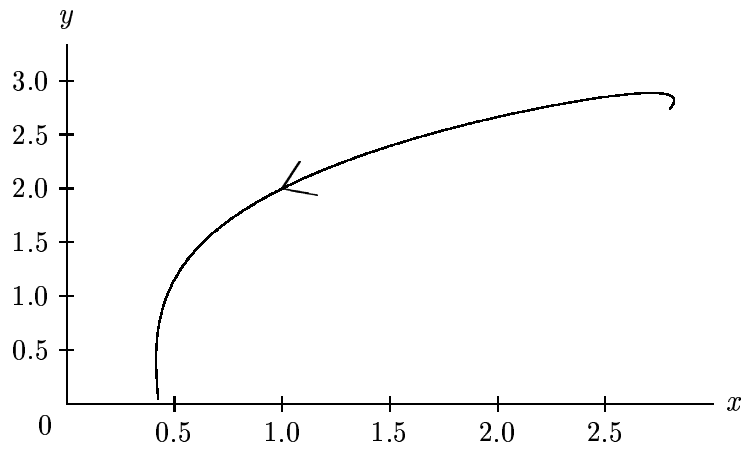
4. (6 points) A particle moves in the xy-plane so that it is at the position $(x(t), y(t))$ at time t, where $x(t)$ and $y(t)$ satisfy the system of differential equations

$$\frac{dx}{dt} = x^2 - y^2\,, \qquad \frac{dy}{dt} = x - 2t\,.$$

It is known that at time $t = 2$, the particle is at the point $(1, 2)$. A graph of the path of the particle is shown in the figure.



Find the instantaneous velocity of the particle at time $t = 2$, and draw an arrow along the curve that shows the direction of motion. *Show your work.*

Substituting $t = 2$, $x = 1$, and $y = 2$ in the given differential equations, we find that when $t = 2$, we have $dx/dt = -3$ and $dy/dt = -3$. Thus at $t = 2$, the particle's instantaneous velocity is

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2} \simeq 4.24\,.$$

Moreover, we can see that the x-position and y-position of the particle are both decreasing (as their derivatives with respect to time are negative). Therefore, at the point $(1, 2)$, **the particle follows the path "downhill"**, *as shown in the figure.*