6. (15 points) For each of the following statements, circle **T** if the statement is always true, and otherwise circle **F**. *You need not explain your answer.*

(a) The formula $1 + x + x^2 + \cdots + x^n = \dfrac{1 - x^{n+1}}{1 - x}$ holds for all real numbers $x \neq 1$ and all positive integers $n = 1, 2, 3, \ldots$.

$\boxed{\mathbf{T}}$ **F**

(b) If $g(x)$ is a periodic function, then every solution $y = f(x)$ of the differential equation $\dfrac{dy}{dx} = g(x)$ is also a periodic function.

T $\boxed{\mathbf{F}}$

(c) If $y = f(t)$ is a solution of the differential equation $\dfrac{dy}{dt} = y^2 - t$, then for every constant C, $f(t) + C$ is also a solution of the differential equation.

T $\boxed{\mathbf{F}}$

(d) The function $y(t) = 0$ is a solution of the initial value problem

$$\frac{dy}{dt} = 3t - y^3\,, \quad y(0) = 0.$$

T $\boxed{\mathbf{F}}$

(e) There is a solution of the logistic differential equation $\dfrac{dP}{dt} = 0.03P\left(1 - \dfrac{P}{3}\right)$ that satisfies $P(1) = 1$ and $P(20) = 5$.

T $\boxed{\mathbf{F}}$