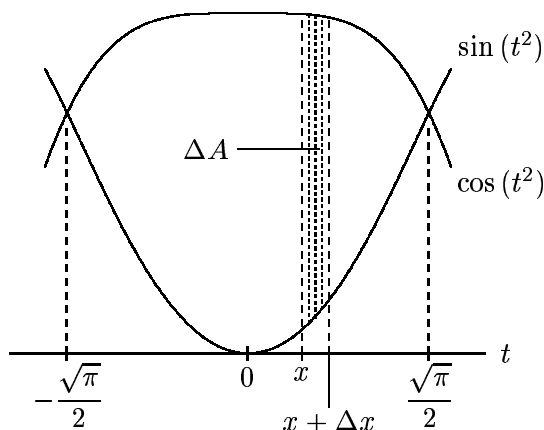


7. (5 points) For  $-\frac{\sqrt{\pi}}{2} \leq x \leq \frac{\sqrt{\pi}}{2}$ , let  $A(x)$  be the area of the region bounded by the curves  $\cos(t^2)$ ,  $\sin(t^2)$ , and the vertical lines  $t = -\frac{\sqrt{\pi}}{2}$  and  $t = x$ . See the figure below.



- (a) Sketch on the figure an area that represents  $\Delta A = A(x + \Delta x) - A(x)$  for a small number  $\Delta x$ .

See the picture above.

- (b) Find a formula for the derivative  $A'(x)$ .

When  $\Delta x$  is very small, the area  $\Delta A$  is well approximated by the area of the box of width  $\Delta x$  and height  $\cos(x^2) - \sin(x^2)$ . Hence we find  $\Delta A \simeq [\cos(x^2) - \sin(x^2)] \Delta x$ .

Thus, we have  $\frac{\Delta A}{\Delta x} \simeq \cos(x^2) - \sin(x^2)$ . Letting  $\Delta x$  go to zero, the ratio  $\frac{\Delta A}{\Delta x}$  approaches  $A'(x)$ , which is how we arrive at the answer given below.

ANSWER :  $A'(x) = \underline{\cos(x^2) - \sin(x^2)}$ .

8. (6 points) For what values of the positive number  $p$  does the infinite series  $\sum_{n=1}^{\infty} \frac{n^3 - 4n^2}{n^p + 5}$  converge? Explain the reason for your answer.

As  $n$  gets very large, the numerator of the general term of the given series “behaves” like  $n^3$ , while its denominator “behaves” like  $n^p$ . Thus, our series is comparable to the series

$$\sum_{n=1}^{\infty} \frac{n^3}{n^p} = \sum_{n=1}^{\infty} \frac{1}{n^{p-3}}.$$

From classwork, we know such a series converges if and only if  $p-3 > 1$ , i.e. if and only if  $p > 4$ .

ANSWER :  $\underline{p > 4}$ .