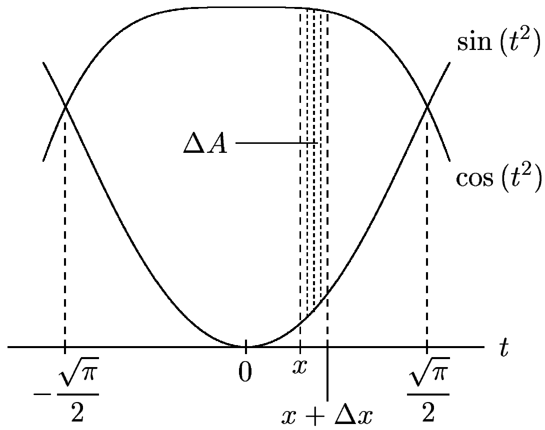
7. (5 points) For $-\frac{\sqrt{\pi}}{2} \leq x \leq \frac{\sqrt{\pi}}{2}$, let $A(x)$ be the area of the region bounded by the curves $\cos(t^2)$, $\sin(t^2)$, and the vertical lines $t = -\frac{\sqrt{\pi}}{2}$ and $t = x$. See the figure below.



(a) Sketch on the figure an area that represents $\Delta A = A(x + \Delta x) - A(x)$ for a small number Δx.

See the picture above.

(b) Find a formula for the derivative $A'(x)$.

When Δx is very small, the area ΔA is well approximated by the area of the box of width Δx and height $\cos(x^2) - \sin(x^2)$. Hence we find $\Delta A \simeq \left[\cos(x^2) - \sin(x^2)\right] \Delta x$.

Thus, we have $\frac{\Delta A}{\Delta x} \simeq \cos(x^2) - \sin(x^2)$. Letting Δx go to zero, the ratio $\frac{\Delta A}{\Delta x}$ approaches $A'(x)$, which is how we arrive at the answer given below.

ANSWER : $\mathbf{A'(x)} = \underline{\cos(\mathbf{x^2}) - \sin(\mathbf{x^2})}$.

8. (6 points) For what values of the positive number p does the infinite series $\sum_{n=1}^{\infty} \frac{n^3 - 4n^2}{n^p + 5}$ converge? *Explain the reason for your answer.*

As n gets very large, the numerator of the general term of the given series "behaves" like n^3, while its denominator "behaves" like n^p. Thus, our series is comparable to the series

$$\sum_{n=1}^{\infty} \frac{n^3}{n^p} = \sum_{n=1}^{\infty} \frac{1}{n^{p-3}}.$$

From classwork, we know such a series converges if and only if $p - 3 > 1$, i.e. if and only if $p > 4$.

ANSWER : $\underline{\mathbf{p > 4}}$.