1. (12 points) The world shrimp production can be represented by the differential equation

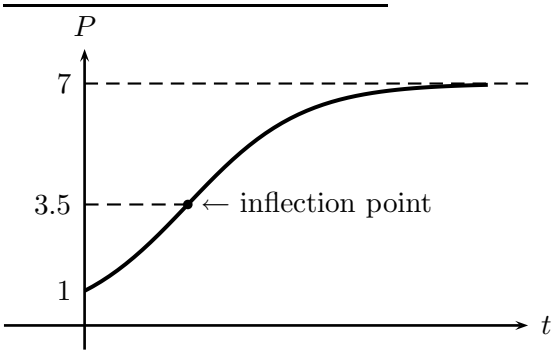
$$\frac{dP}{dt} = -0.1P(P-7),$$

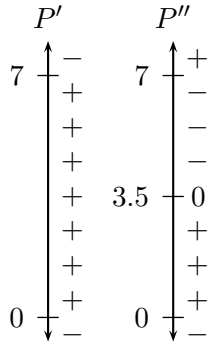
where t is the number of years since 1982 and $P(t)$ is the quantity of shrimp farmed in the world during year t in hundreds of thousands of metric tons. In 1982 the world shrimp production was 100,000 metric tons.

(a) (3 pts.) Determine all of the equilibrium solutions of the differential equation given above. Classify each as either stable or unstable. No explanation required.

- $P = 0$ (unstable equilibrium;)
- $P = 7$ (stable equilibrium.)

(b) (4 pts.) Sketch a graph of the solution to the given initial value problem. Be sure to indicate clearly on your graph where the solution curve is increasing/decreasing and where it is concave up/concave down. Clearly mark the value of any asymptotes.





Note

$$P''(t) = \frac{d}{dt} - 0.1P(P-7)$$
$$= -0.1(2P-7)\frac{dP}{dt}$$
$$= .01P(P-7)(2P-7).$$

(c) (3pts.) Use Euler's method with $\Delta t = 0.5$ to estimate the world shrimp production in the year 1984 ($t = 2$).

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t_i	P_i	slope at (t_i, P_i)	$\Delta P_i = \Delta t \times$ (slope at (t_i, P_i))
0.0	1.000	0.600	0.300
0.5	1.300	0.741	0.371
1.0	1.671	0.890	0.445
1.5	2.116	1.033	0.517
2.0	**2.632**		

So, the world's shrimp production in 1984 was approximately 263,200 metric tons.

(d) (2 pts.) Is the estimate of world shrimp production in part (c) bigger or smaller than the exact solution to the initial value problem at $t = 2$? Explain in one sentence.

The estimate is smaller than the actual value. The exact solution curve is concave up when P is between 1 and 2.632, so a tangent line-based approximation to the actual solution curve yields an underestimate to the actual values.