2. (10 points) An apple is placed in a room whose air temperature is fixed at 50 °F. Let $T(t)$ be the temperature of the apple at time t, which is measured in hours. According to Newton's Law of Heating and Cooling, the rate of change of the apple's temperature satisfies

$$\frac{dT}{dt} = k(T - 50).$$

(a) (2 pts.) What is the value of k if the temperature decreases at an instantaneous rate of 3 °F per hour when the temperature T of the apple is 65?

$$\frac{dT}{dt} = -3, \text{ when } T = 65, \quad \text{so}$$

$$-3 = k\ (65 - 50), \quad \text{or } k = -0.2.$$

(b) (5 pts.) Now assume $k = -0.1$. Solve the initial value problem $dT/dt = k(T - 50)$ with $T(0) = 30$.

Separating variables, we have

$$\int \frac{1}{T-50}\ dT = \int k\ dt$$

$$\ln |T - 50| = kt + C$$

$$T = Ae^{kt} + 50; \quad \text{where } A \text{ is an arbitrary constant.}$$

Now, using the initial condition (0,30), we get $30 = A + 50$ or $A = -20$.
So,

$$T = 50 - 20e^{-0.1t}.$$

(c) (3 pts.) Briefly explain what the solution from part (b) says about the temperature of the apple over the time interval $[0, \infty)$.

The apple progressively warms up (initially rather fast, then slower and slower) from its initial temperature of 30° F to just about 50° F in the long run (as this is the temperature of the surrounding air.