5. (12 points) Determine whether each of the following series converges or diverges. Circle CONVERGES or DIVERGES and then BRIEFLY EXPLAIN why each series converges or diverges. In each part of the problem you will receive one point for circling the correct answer (and only the correct answer) and up to two points for your explanation.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n+2}$ $\boxed{\textsc{Diverges}}$ $\textsc{Converges}$

Explanation:

Since $\displaystyle\lim_{n\to\infty} \frac{n+1}{n+2} = 1 \neq 0$, the terms of the series do not approach zero, which means that their infinite sum diverges.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^5+2}$ $\textsc{Diverges}$ $\boxed{\textsc{Converges}}$

Explanation:

For large n, $n^3/(n^5+2) \simeq n^3/n^5 \simeq 1/n^2$. So the given series behaves like $\sum 1/n^2$, which converges by the integral test.

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $\boxed{\textsc{Diverges}}$ $\textsc{Converges}$

Explanation:

Since $\int 1/(x \ln x)\ dx = \int 1/u\ du = \ln|u| + C = \ln|\ln x| + C$, and $lim_{b\to\infty} \ln|\ln b| = \infty$, the sum diverges by the integral test.

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$ $\textsc{Diverges}$ $\boxed{\textsc{Converges}}$

Explanation:

Using the ratio test,

$$\lim_{n\to\infty} \left[\frac{(n+1)^2 2^{n+2}}{3^{n+1}} \frac{3^n}{n^2 2^{n+1}} \right] = \lim_{n\to\infty} \frac{2(n+1)^2}{3n^2} = 2/3.$$

Since $2/3 < 1$, we have convergence.