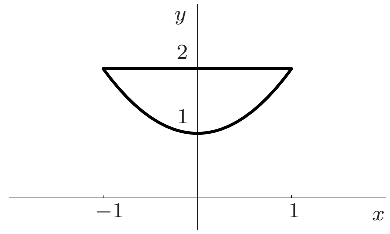
6. (18 points) A thin metal plate lying in the region bounded by the line $y = 2$ and the parabola $y = x^2 + 1$ has uniform density 5 gm/cm^2.



(a) (2 pts.) Write an integral expression giving the exact *area* of this region. *Do not* evaluate this expression.

$$4 - 2\int_0^1 (1 + x^2)\, dx$$

(b) (4 pts.) Write an integral expression giving the exact *perimeter* of this region. *Do not* evaluate this expression.

$$2 + 2\int_0^1 \sqrt{1 + 4x^2}\, dx$$

(c) (5 pts.) Write a definite integral giving the exact *volume* of the solid generated by rotating the region about the x-axis. *Do not evaluate this integral*

$$2\pi \int_0^1 [4 - (1 + x^2)^2]\, dx$$

(d) (7 pts.) Find the coordinates of the *center of mass* for this metal plate. Show your work.

- $\overline{x} = 0$ (as the plate is symmetric about the y-axis)
- $\overline{y} = \dfrac{\text{Moment}}{\text{Mass}} \simeq \dfrac{10.667}{6.667} \simeq 1.6$ (units above the x-axis),

since

- Moment $= \displaystyle\int_1^2 5y(2\sqrt{y-1})\, dy = 10\int_1^2 y\sqrt{y-1}\, dy \simeq 10.667$;
- Mass $= \displaystyle\int_1^2 5(2\sqrt{y-1})\, dy = 10\int_1^2 \sqrt{y-1}\, dy \simeq 6.667$.