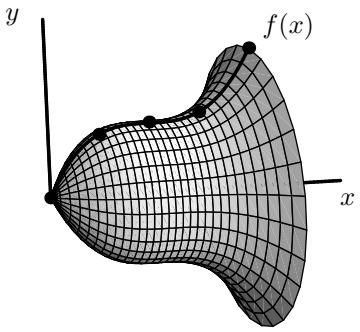
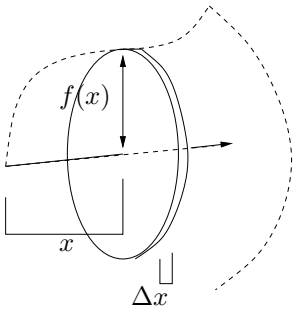
1. [11 points] Consider the shape shown to the right. The function shown as a dark curve is $f(x)$. The points on the curve are the points $(0, f(0))$, $(0.5, f(0.5))$, $(1, f(1))$, $(1.5, f(1.5))$, and $(2, f(2))$.



(a) [4 points of 11] Draw a slice, below, that you might use to find the total volume enclosed by the shape if you were to be doing this by integration. Label in your figure x, $f(x)$, Δx, and any other relevant quantities.

Solution:
We are given the function $f(x)$, which provides the radius of circular cross-sections of the object shown. Thus we slice vertically, getting slices that are disks, as shown in the figure below.



(b) [2 points of 11] Write an integral giving the volume of the shape.

Solution:
The volume of the slice is $\Delta V = \pi\,(f(x))^2\,\Delta x$, so that, letting Δx go to zero, we can sum all such slices with an integral. The resulting volume is $V = \int_0^2 \pi\,(f(x))^2\,dx$.

(c) [5 points of 11] If the points shown in the figure are, in order from left to right, (0,0), (0.5,0.875), (1,1), (1.5,1.125) and (2,2), estimate the volume using the trapezoid method.

Solution:
Left- and right-hand sums for the volume are

$$\text{Left} = (0.5)(\pi(0)^2 + \pi(0.875)^2 + \pi(1)^2 + \pi(1.125)^2) \approx 4.761$$
$$\text{Right} = (0.5)(\pi(0.875)^2 + \pi(1)^2 + \pi(1.125)^2 + \pi(2)^2) \approx 11.04$$

Then the trapezoid method gives $\text{Trap} = \frac{1}{2}(4.761 + 11.04) = 7.90$.