

10. [7 points] On the morning of December 1, Alex belatedly considered buying a plane ticket to go home the winter break. Unhappy with the price of the ticket, Alex decided to wait and see if the prices would drop at all, but, by checking with friends, determined that the rate of change in the price on December 1 was \$1.58/day. In the next two weeks, between days spent studying for calculus, Alex noted the rate of change in price, finding the data in the table below.

Date	Dec. 1	Dec. 4	Dec. 7	Dec. 10	Dec. 13	Dec. 16
Rate of Price Change (\$/day)	1.58	3.86	5.62	8.24	13.00	20.72

Admitting defeat, Alex bought a plane ticket on December 16. Give a good estimate for how much money Alex could have saved by buying the plane ticket on December 1.

Solution:

The total change in the price of the plane ticket is $\int_1^{16} (\text{rate of change}) dt$. We can estimate this integral using a left- or right-hand sum. These give

$$\text{Left sum} = (3)(1.58 + 3.86 + 5.62 + 8.24 + 13) = \$96.90$$

$$\text{Right sum} = (3)(3.86 + 5.62 + 8.24 + 13 + 20.72) = \$154.32.$$

Because it appears that the rate is an increasing function, we expect that the left sum is an underestimate and the right an overestimate, so that a better guess would be the trapezoid estimate $\text{Trap} = \frac{1}{2}(96.90 + 154.32) = \125.61 .