11. [12 points] For each of the following, sketch a graph or figure as indicated. (There is more than one correct answer for each.) Your sketches do not have to be detailed, but should clearly illustrate the characteristics described.

(a) [3 points of 12] Sketch a non-constant function $f(x)$ such that neither of the left- or right-hand estimates for $\int_a^b f(x) \, dx$ are overestimates.

Solution:

Or, e.g.,

(b) [3 points of 12] Sketch the antiderivative of a function $f(x)$, on the interval $a \leq x \leq b$, if $\int_a^b f(x) \, dx < 0$.

Solution:

Key: $F(b)$ must be less than $F(a)$.

(c) [3 points of 12] Sketch a function given in polar coordinates as $r = f(\theta)$, and the area represented by $\frac{1}{2} (f(\pi/4))^2 \Delta \theta$.

Solution:

(d) [3 points of 12] Sketch a sequence $S_n$ of partial sums for a convergent series $\Sigma a_n$ if $\Sigma a_n = L$.

Solution:

Other figures are also ok, e.g.,