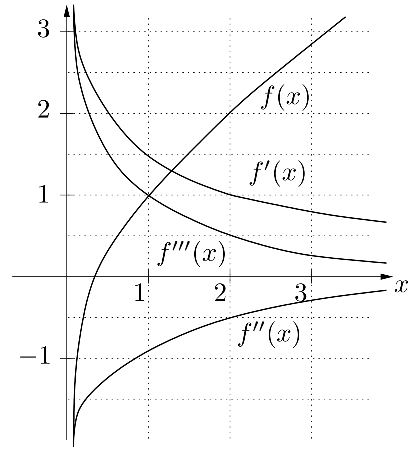
2. [7 points] The graph to the right shows $f(x)$, $f'(x)$, $f''(x)$ and $f'''(x)$.



(a) [4 points of 7] Find the 3rd degree Taylor polynomial approximating $f(x)$ near $x = 2$.

Solution:
We know that the 3rd degree Taylor polynomial is $P_3 = f(2) + f'(2)(x-2) + \frac{1}{2!}f''(2)(x-2)^2 + \frac{1}{3!}f'''(2)(x-2)^3$. We can read the values for f and its derivatives from the graphs, finding $f(x) = 2$, $f'(2) = 1$, $f''(2) = -\frac{1}{2}$ and $f'''(2) = \frac{1}{2}$. Thus

$$P_3 = 2 + (x-2) - \frac{1}{4}(x-2)^2 + \frac{1}{12}(x-2)^3.$$

(b) [3 points of 7] Based on the graphs of f and its derivatives that you have in the given figure, what would you guess the radius of convergence of the Taylor expansion for $f(x)$ around $x = 2$ would be? Explain.

Solution:
From the graphs it is clear that f and its derivatives have a vertical asymptote at $x = 0$. It is not possible for a polynomial expansion to reproduce this, so we would expect that the Taylor expansion would fail there. This is two units from $x = 2$, so we guess that the radius of convergence is $R = 2$.