4. [10 points] Consider the solution curve $y(x)$ to the differential equation $\frac{dy}{dx} = 1 + xy^2$ that goes through the point $(1, 1)$.

(a) [2 points of 10] Is this differential equation separable? Explain in one or two sentences.

Solution:
This is not separable. In order to separate variables we have to be able to get all of the "y"s on one side of the equation, multiplying the dy, and all of the "x"s on the other, multiplying the dx. Because of the additive factor of one on the right-hand side this is not possible.

(b) [5 points of 10] Use Euler's Method with three steps to approximate y when $x = 1.6$.

Solution:
If we're taking three steps, our step size is $\Delta x = 0.2$. At $(x, y) = (1, 1)$ the slope is $\frac{dy}{dx} = 2$, so we estimate $y(1.2) \approx 1 + 0.2(2) = 1.4$. Continuing, we obtain the following table of values:

x	y	dy/dx	so that
1	1	2	$y(1.2) \approx 1 + 0.2(2) = 1.4$
1.2	1.4	$1 + (1.2)(1.4)^2 = 3.352$	$y(1.4) \approx 1.4 + 0.2(3.352) = 2.0704$
1.4	2.0704	$1 + (1.4)(2.0704)^2 \approx 7.001$	$y(1.6) \approx 2.0704 + 0.2(7.001) \approx 3.471$

Thus, we estimate that $y(1.6) \approx 3.471$.

(c) [3 points of 10] Do you expect the real value of $y(1.6)$ to be greater than or less than the estimate you found in part (b)? Why?

Solution:
We note that the slopes that we generated in the approximations (column 3 in the table above) are increasing. We therefore expect that the Euler estimates will undershoot the actual values at every step, and that the estimate $y(1.6) \approx 3.471$ is an underestimate.