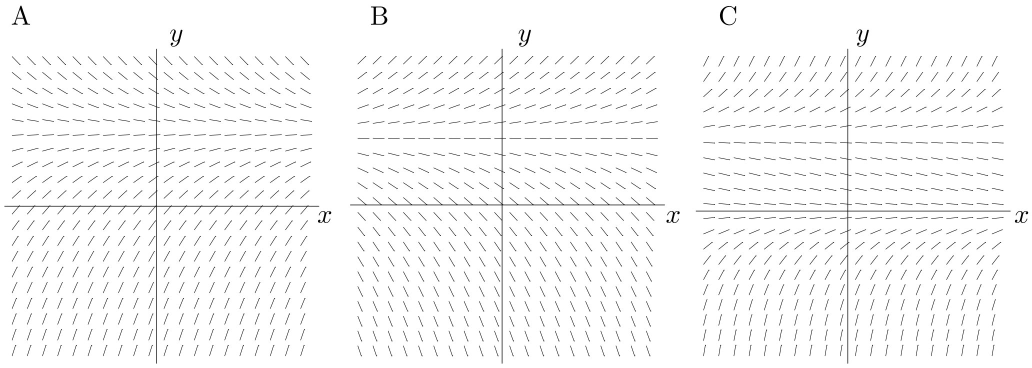
6. [8 points] Shown below are the slope fields for three differential equations, "A," "B," and "C." For each slope field, the axes intersect at the origin.



For each of the following functions, indicate which, if any, of the differential equations "A," "B," and "C" it could be the solution of. Note that any given function may be the solution to zero, one, or more than one of the differential equations. If a function is a solution to none of these differential equations, clearly write "None" as your answer.

(a) $y = 0$

Solution:
Answer: "C." This is a constant, so any differential equation(s) it solves must have the equilibrium solution $y = 0$. This means that the slope is zero for any x when $y = 0$, which is true only for differential equation "C." Thus $y = 0$ is a solution only to "C."

(b) $y = 1$

Solution:
Answer: "A, B, C." Again, this is a constant, so any differential equation(s) it solves must have the equilibrium solution $y = 1$. This means that the slope is zero for any x when $y = 0$, which is true for all three differential equations—if the horizontal slopes in the figures are at $y = 1$. Thus $y = 1$ could be a solution to any of "A," "B," or "C."

(c) $y = 1 + ke^x$

Solution:
Answer: "B." We note that if $k = 0$ we get the constant solution $y = 1$. If $k < 0$, we must be starting with the initial condition $y(0) < 1$. For all such cases, the exponential grows and as $x \to \infty$, $y \to -\infty$. We can see that for differential equation "B" this is the case: following the slope field lines from any point on the y axis below $y = 1$ gives a solution that decreases faster and faster. For "A" and "C" this is not the case—while "C" has decreasing solutions for $0 < y(0) < 1$, for $y(0) < 0$ they increase. Thus this solution could solve "B" only.

(d) $y = 1 + ke^{-x}$

Solution:
Answer: "A." As in the previous problem, if $k = 0$ we get the constant solution $y = 1$. If $k < 0$, we must be starting with the initial condition $y(0) < 1$. For all such cases, the exponential decays to zero and as $x \to \infty$, $y \to 1$. We can see that for differential equation "A" this is the case: following the slope field lines from any point on the y axis below $y = 1$ gives an increasing solution that approaches one. For "B" and "C" this is not the case. Thus this could solve "A" only.