7. [6 points] As part of a final project in a chemistry class, Alex is studying a reaction that combines a small amount of catalyst with a large amount of another reagent. The lab manual indicates that if the amount of reagent used is $R+x$, where R is the (large) intended amount and x is a small variation from that, then the amount of catalyst required is $c(x) = k\sqrt{R+x}$. However, Chris thinks that it would be reasonable (and easier!) to use $c(x) = k\sqrt{R}(1+\frac{x}{2R})$ instead.

(a) [4 points of 6] Are Chris' and the lab book's expressions consistent? Explain. (Hint: your answer should not involve graphing.)

Solution:
They are consistent (of course). If we start with the lab book's expression and factor $\sqrt{R}$ out of the square root, we get $c(x) = k\sqrt{R}\sqrt{1+\frac{x}{R}}$. Then $\frac{x}{R}$ is small, so we can logically expand this as a Taylor series for small $\frac{x}{R}$ with the binomial expansion. This gives $c(x) = k\sqrt{R}\,(1+\frac{x}{2R}-\frac{x^2}{8R^2}+\cdots)$. Thus the Chris' expression is the same as the lab book's up to the $\frac{x}{R}$ term.

(b) [2 points of 6] Assuming that the two expressions are consistent, is Chris' estimate an over- or underestimate of the actual amount of catalyst required? Why?

Solution:
The next term in the binomial expansion is $-\frac{x^2}{8R^2}$, which decreases the value of $c(x)$ (and, because R is large, subsequent terms are smaller), so Chris' estimate is an overestimate.