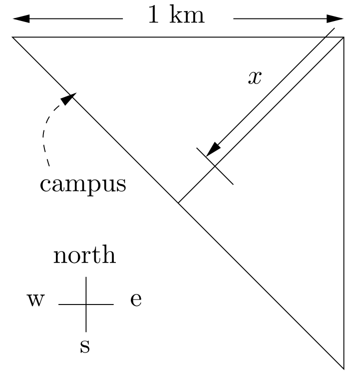
8. [10 points] It turns out that there are a fair number of squirrels on the campus of Alex and Chris' university. The university occupies a triangular piece of land that is half of a 1 km by 1 km square, as shown in the figure to the right. Owing to the proximity of a local arboretum, the population of squirrels is densest at the northeast corner of campus. In addition, Alex has noted that if x is the distance measured along a line running diagonally northeast-southwest across the campus, as shown in the figure, the population density of squirrels everywhere along a line perpendicular to the diagonal is given by $p(x) = \frac{100}{x(1+x)}$ squirrels per square meter. How many squirrels are there on campus? (Note that there are 1000 meters in a kilometer.)



Solution:
Because the squirrel population is constant along the perpendicular lines, we want to slice the campus into slices along these perpendiculars. Each slice has width Δx, and is a distance x along the indicated northwest–southeast line. Note that $0 \le x \le \frac{1000}{\sqrt{2}}$. Then the length of the slice is $2x$, and its area is $2x\,\Delta x$. The number of squirrels on the slice is then $\rho(x) \cdot 2x\,\Delta x = \frac{200}{1+x}\,\Delta x$. We can find the total number of squirrels by integrating this over all possible values of x, finding

$$\#\text{ squirrels} = \int_0^{1000/\sqrt{2}} \frac{200}{(1+x)}\,dx = 200\,\ln(1+x)\Big|_0^{1000/\sqrt{2}} = 200\,\ln(1+\frac{1000}{\sqrt{2}}) \approx 1312.52.$$

A fractional squirrel seems a bit morbid, so let's assume that there are 1313 squirrels.