10. [8 points] Each of the following statement is either False (there are counter-examples to the statement), True, or True if a condition holds. For each, circle the correct characterization (obviously, a True statement is also True if the condition holds; circle "True" in this case, not "True if…"). No explanations are necessary.

(a) [2 points of 8] $y = 3x^2$ is a solution to $xy' = 2y - b$

True False **[True if $b = 0$]**

Solution:
True if $b = 0$: $y' = 6x$, so $xy' = 6x^2 = 2(3x^2) = 2y$.

(b) [2 points of 8] $\int_{-1}^{1} \frac{1}{1+kx^2}\,dx$ is an improper integral.

True False **[True if $k \le -1$]**

Solution:
True if $k \le -1$: If $k > -1$ there is no singularity in the denominator and the integral is proper.

(c) [2 points of 8] If $F'(x) = x\sin(e^x)$, then $F(x) = \int_0^\infty t\sin(e^t)\,dt$.

True **[False]** True if $F(0) = 0$

Solution:
False: if $F'(x) = x\sin(e^x)$ we can write $F(x) = \int_c^x t\sin(e^t)\,dt$, and if we also know $F(0) = 0$, then $F(x) = \int_0^x t\sin(e^t)\,dt$. But neither of these is the same as $\int_0^\infty t\sin(e^t)\,dt$.

(d) [2 points of 8] $F(t) = \begin{cases} 0, & t < 0 \\ t/a, & 0 \le t < a \\ 1, & t \ge a \end{cases}$ could be a cumulative distribution function.

[True] False True if $a = 1$

Solution:
True: $F(t) = 0$ at the left end of its domain and 1 at the right end, and is never decreasing.