4. [12 points] Newton's law of cooling (or warming) says that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Suppose that a thermometer used by a veterinarian to find the temperature of a sick horse obeys Newton's law of cooling. Further suppose that before insertion the thermometer reads 82° F, after one minute it reads 92° F, and after another minute it reads 97° F, and that a sudden convulsion unexpectedly destroys the thermometer after the 97° reading. Call the horse's temperature T_h.

(a) [3 points of 12] Write a differential equation for the temperature T (a function of time t) of the thermometer. Your equation may involve the constant T_h.

Solution:
We know that $\frac{dT}{dt}$ is proportional to the temperature difference between thermometer and horse, so

$$\frac{dT}{dt} = k(T_h - T),$$

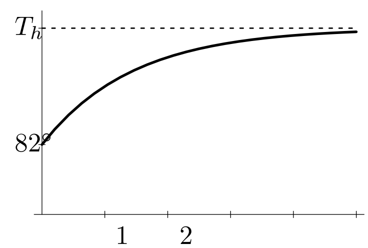
where T_h is the horse's temperature.

(b) [3 points of 12] Solve the differential equation for T to find a general solution for T. Your solution may include undetermined constants such as T_h.

Solution:
Separating variables, we have $\frac{dT}{T-T_h} = -kdt$, so that $\ln|T - T_h| = -kt + A$. Exponentiating and setting $C = \pm e^A$, we find $T = T_h + Ce^{-kt}$.

(c) [3 points of 12] Sketch a graph of T, indicating the initial temperature and T_h on your graph.

Solution:
We know that T is increasing, is an exponential function, and that it decays towards T_h. Further, we know that the time scale should be in minutes because of the initial data given. This leads to the graph shown to the right.



(d) [3 points of 12] Write a set of equations that would allow you to determine the horse's temperature (and the other undetermined constants in your expression for T). *Do not solve these equations.*

Solution:
When $t = 0$, $T = 82$, so $T = T_h + C = 82$; when $t = 1$, $T = T_h + Ce^{-k} = 92$; and when $t = 2$, $T = T_h + Ce^{-2k} = 97$. This gives us a set of equations for T_h, k and C,

$$\begin{aligned} T_h + C &= 82 \\ T_h + Ce^{-k} &= 92 \\ T_h + Ce^{-2k} &= 97, \end{aligned}$$

that would allow us to solve for T_h.