

4. [12 points] The density of the Earth changes with the distance below the surface of the Earth one goes. If  $x$  gives the distance (in km) below the surface, the density  $\delta(x)$  (in  $\text{kg}/\text{km}^3$ ) is approximately

$x$	0	1000	2000	2900	3000	4000	5000	6000	6370
$\delta(x)$	3300	4500	5100	5600	10,100	11,400	12,600	13,000	13,000

(the radius  $R$  of the Earth is about 6370 km). Let  $r$  measure the distance out from the center of the Earth.

- a. [4 points] The integral  $\int_0^{4000} (4\pi \cdot r^2 \cdot \delta(R-r)) dr$  is the limit as  $\Delta r \rightarrow 0$  of a Riemann sum  $\sum 4\pi \cdot r^2 \cdot \delta(R-r) \cdot \Delta r$ . In the context of this problem, what do the terms of this sum represent?

- b. [4 points] Now consider the integral  $\int_{R-4000}^R (4\pi \cdot r^2 \cdot \delta(R-r)) dr$ . Rewrite this in terms of the variable  $x$ . Estimate your rewritten integral with MID(2).

- c. [4 points] Let  $F(x) = \int_{R-x}^R (4\pi \cdot r^2 \cdot \delta(R-r)) dr$ . Find  $F'(x)$ , showing work that shows how you obtained your answer.