1. [10 points] Suppose that the first and third degree Taylor polynomials, \( P_1(x) \) and \( P_3(x) \), approximating a function \( g(x) \) at \( a = 0 \) are given in the graph to the right, below.

a. [6 points] Using these Taylor polynomials, what are \( g(0) \) and \( g'(0) \)? What is the sign of \( g''(0) \)?

Solution: We know \( g(0) = 1 \), and from \( P_1(x) \), \( g'(0) = 1 \). Then \( P_3(x) \) appears to be concave down, so we know that \( g''(0) < 0 \). We cannot, however, determine a value for \( g''(0) \).

b. [4 points] Could \( g(x) \) be the function \( 1 + \sin(x) \)? Why or why not?

Solution: For this \( g(x) \) we know \( g(0) = 1 \), \( g'(0) = \cos(0) = 1 \) and \( g''(0) = -\sin(0) = 0 \). While the first two of these are consistent with our observations in (a), the last is not, so \( g(x) \) could not be the function \( 1 + \sin(x) \).