2. [10 points] Find the interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n\,(x-1)^n}{n}$.

Solution: To find the radius of convergence, we use the ratio test:

$$\lim_{n\to\infty}\left|\frac{2^{n+1}\,(x-1)^{n+1}}{n+1}\cdot\frac{n}{2^n\,(x-1)^n}\right| = |x-1|\lim_{n\to\infty}\frac{2n}{n+1} = 2|x-1|.$$

This must be less than one, so $|x-1|<\frac{1}{2}$, and the radius is $R=\frac{1}{2}$. Testing convergence at the endpoints, we have at $x=\frac{1}{2}$ the series $\sum \frac{2^n\,(-\frac{1}{2})^n}{n} = \sum \frac{(-1)^n}{n}$, the alternating harmonic series, which converges. At $x=\frac{3}{2}$, we similarly have $\sum \frac{2^n\,(\frac{1}{2})^n}{n} = \sum \frac{1}{n}$, which is the harmonic series, which diverges. Thus the interval of convergence is $\frac{1}{2}\le x<\frac{3}{2}$.