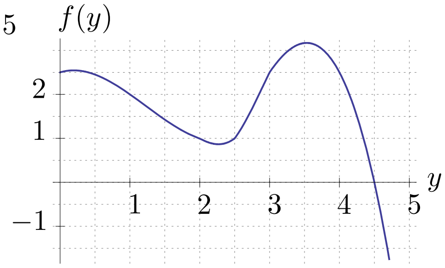
3. [16 points] Suppose that $\frac{dy}{dt} = f(y)$, where $f(y)$ is given by the graph in the figure to the right, below.

a. [4 points] If $y(0) = 1$, use Euler's method with $\Delta t = 0.5$ to estimate $y(1)$.



Solution: Using Euler's method, we approximate

$$y(0.5) \approx y(0) + 0.5\, f(y(0)) = 1 + 0.5(2) = 2,$$

and

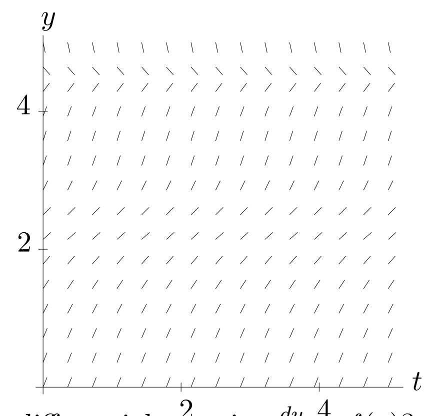
$$y(1) \approx y(0.5) + 0.5\, f(y(0.5)) = 2 + 0.5(1) = 2.5.$$

b. [4 points] Could $y(t) = 2.5 - t^2$ be a solution to the given differential equation $\frac{dy}{dt} = f(y)$? Why or why not?

Solution: No; $\frac{dy}{dt} = -2t = \mp 2\sqrt{2.5 - y}$, which clearly could not generate the given graph.
Alternately, note that if we start at $(0, 1)$ we know $\frac{dy}{dt} = f(1) = 2$, but if $y = 2.5 - t^2$, $\frac{dy}{dt}\Big|_{t=0} = 0 \neq 2$.

c. [4 points] Could the slope field given to the right, below, be the slope field for the given differential equation $\frac{dy}{dt} = f(y)$? Why or why not?

Solution: This could be the indicated slope field; it depends only on y, and the slopes at different y values appear to be similar to the function values $f(y)$ shown in the figure above.



d. [4 points] Are there any equilibrium solutions to the given differential equation $\frac{dy}{dt} = f(y)$? If so, are they stable? If there are none, why are there none?

Solution: Yes, $y = 4.5$ is an equilibrium solution. It is stable, because for values of $y < 4.5$ the slope is positive, while for values $y > 4.5$ the slope is negative.