3. [16 points] Suppose that \( \frac{dy}{dt} = f(y) \), where \( f(y) \) is given by the graph in the figure to the right, below.

a. [4 points] If \( y(0) = 1 \), use Euler’s method with \( \Delta t = 0.5 \) to estimate \( y(1) \).

Solution: Using Euler’s method, we approximate
\[
y(0.5) \approx y(0) + 0.5 f(y(0)) = 1 + 0.5(2) = 2,
\]
and
\[
y(1) \approx y(0.5) + 0.5 f(y(0.5)) = 2 + 0.5(1) = 2.5.
\]

b. [4 points] Could \( y(t) = 2.5 - t^2 \) be a solution to the given differential equation \( \frac{dy}{dt} = f(y) \)? Why or why not?

Solution: No; \( \frac{dy}{dt} = -2t = \mp 2\sqrt{2.5 - y} \), which clearly could not generate the given graph.
Alternatively, note that if we start at (0,1) we know \( \frac{dy}{dt} = f(1) = 2 \), but if \( y = 2.5 - t^2 \), \( \frac{dy}{dt} \big|_{t=0} = 0 \neq 2 \).

c. [4 points] Could the slope field given to the right, below, be the slope field for the given differential equation \( \frac{dy}{dt} = f(y) \)? Why or why not?

Solution: This could be the indicated slope field; it depends only on \( y \), and the slopes at different \( y \) values appear to be similar to the function values \( f(y) \) shown in the figure above.

d. [4 points] Are there any equilibrium solutions to the given differential equation \( \frac{dy}{dt} = f(y) \)?
If so, are they stable? If there are none, why are there none?

Solution: Yes, \( y = 4.5 \) is an equilibrium solution. It is stable, because for values of \( y < 4.5 \) the slope is positive, while for values \( y > 4.5 \) the slope is negative.