

5. [14 points] A water filtration system has a rectangular basin 0.2 m wide, 0.4 m long and 0.2 m deep in which unfiltered water is poured. Along the bottom of the basin there is a filter, in the shape of a square, measuring 0.01 m on each side.
- a. [3 points] The density of water is 1000 kg/m^3 , and the gravitational constant is 9.8 m/sec^2 . Suppose the depth of the water in the basin is h m. What is the force due to water pressure on the filter? Include units in your answer.
- b. [3 points] Write an equation for the volume of water in the basin, V , when the water is h m deep. Then use this equation to write an equation for the rate of change of volume of water, $\frac{dV}{dt}$, in terms of the rate of change of depth of water, $\frac{dh}{dt}$.
- c. [5 points] The rate at which water passes out of the basin is proportional to the square of the force due to water pressure exerted on the filter, with constant of proportionality $k > 0$. Suppose the basin is filled with water at a constant rate $r \text{ m}^3/\text{sec}$. Write a differential equation for the depth h of the water in the basin. That is, find an equation for $\frac{dh}{dt}$ in terms of h and constants r and k .
- d. [3 points] Suppose when the basin is filled at a constant rate of $0.0002 \text{ m}^3/\text{sec}$, the depth remains constant at 0.1 m. What is the constant of proportionality k ?